Theory of measuring the "Luttinger-g" of a one-dimensional quantum dot

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We study electron transport through a quantum dot in a Tomonaga-Luttinger liquid with an inhomogeneity induced either by a non-uniform electron interaction or by the presence of tunnel resistances of contacts. The non-analytic temperature behavior of the conductance peaks show crossovers determined by the different energy scales associated with the dot and the inhomogeneity despite the Coulomb blockade remains intact. This suggests an explanation of recent findings in semiconductor quantum wires and carbon nanotubes.

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One dimensional (1D) electron systems are important paradigms for studying the effects of impurities and interactions in condensed matter. Here, electron-electron interaction can be treated by the bosonization technique using the Tomonaga-Luttinger liquid (TLL) model. The energetically lowest excitations are collective charge- and spin-density waves [1]. Correlation functions can be exactly obtained. As a function of temperature, frequency and/or bias voltage, non-analytic power laws of the form $C(\tau) \propto \tau^{\nu(g)}$ have been predicted, where $\nu(g)$ is a non-rational exponent that contains the interaction parameter g. Since TLL appears to be of fundamental importance in modern condensed matter physics, experimentally confirming such behavior is very important.

The search for "Tomonaga-Luttinger behavior" has been very intense during recent years. In tunable semiconductor quantum wires contacted to nearly adiabatic funnels [2] conductance quantization was found to be weakly affected by electron correlations. It was argued that in these systems the conductance of the leads (assumed as Fermi liquids) is measured [3]. Non-universal conductance quantization was detected in semiconductor cleaved-edge-overgrowth (CEO) quantum wires [4]. This was assigned to electron backscattering in the contacts [5]. Theoretical approaches considered contacts and impurity scattering [6–8]. In recent CEO-experiments, a 1D quantum dot was fabricated between two impurities [9]. The dependence on temperature T of Coulomb blockade peaks was cleanly analyzed within the TLL model [10–12]. Most strikingly, interaction parameters deduced from different measured quantities were found to be inconsistent: from the charging energy E_c, g was estimated to be a factor of two smaller than the value obtained from the temperature dependence of the conductance peaks. Similar inconsistencies were observed in the temperature behavior of the conductance of a carbon nanotube (CN) SET-transistor formed between two buckles [13].

In this paper, we consider a 1D quantum dot in a TLL with an inhomogeneity induced either by a non-uniform interaction parameter or by the presence of contacts modeled by tunnel resistances. First, in view of the CEO-system, we determine the conductance for $g(x)$ interpolating smoothly between $g_\infty = g(|x| \to \infty)$ and $g_b = g(x_b)$ in the dot. We find that the interaction in different regions of the system can be probed. While E_c and the level spacing ε in the dot are determined by the interaction near x_b, the temperature behavior of the conductance peaks show crossovers between g_∞ (low T) and g_b (high T). Second, we determine the conductance of a quantum dot embedded in a TLL with $g = \text{const}$ but attached to metallic contacts. This is appropriate for CNs. We show that the temperature behavior of the Coulomb peaks can be entirely determined by the non-analytic powers laws induced by the contacts though the Coulomb blockade peaks remain perfectly intact. Quite generally, the crossovers can be traced back to the competing energy scales associated with the quantum dot and the inhomogeneity. Our results not only put earlier theoretical findings into a general frame but also explain quantitatively the — at the first sight — discrepant findings in the recent experiments which seems to us of outstanding importance after the intense search for TLL-behavior during more than three decades.

We consider the Hamiltonian $H = H_0 + H_b$ with

$$H_0 = \frac{\hbar v_F}{2} \int \mathrm{d}x \left\{ \Pi^2(x) + \frac{1}{g^2(x)} [\partial_x \vartheta(x)]^2 \right\} \qquad (1)$$

describing the clean TLL-system with the conjugate Boson fields $[\Pi(x), \vartheta(x')] = i\delta(x - x')$ and a local inhomogeneous density-density interaction

$$\frac{1}{g^2(x)} = \frac{1}{g_\infty^2} + \frac{\varphi(x)}{\pi \hbar v_F}. \qquad (2)$$

We assume φ as smooth, with a maximum of width $L^* \gg a$ near the impurities, and $\varphi(x) \to 0$ when $|x| \to \infty$.

The contribution of the impurities located at $x_b^\pm = x_b \pm a/2$ is $H_b = U_b \cos(\pi N_+) \cos[\pi(n_0 + N_-)]$ where

$N_\pm = [\vartheta(x_{\rm b}^+) \pm \vartheta(x_{\rm b}^-)]/\sqrt{\pi}$. The quantity N_+ and N_- are associated with the unbalanced particles between left and right leads, and the fluctuations of the particle number in the dot with respect to the mean value $n_0 = k_{\rm F} a/\pi$, respectively ($k_{\rm F}$ Fermi wave number). The charge modes of the inhomogeneous TLL act on $N_\pm$ as an external Bosonic bath. Their influence can be exactly evaluated in terms of a dissipative impedance [12]. For $L^* > a$, such that near the quantum dot $g(x) \approx g_{\rm b}$, the resulting Euclidean effective action is

$$S_{\rm eff}[N_+, N_-] = \frac{k_{\rm B}T}{2} \sum_{r=\pm,n} N_r(\omega_n) K_r(\omega_n) N_r(\omega_n). \quad (3)$$

The Fourier transforms of the dissipative kernels K_r, at Matsubara frequencies $\hbar\omega_n = 2k_{\rm B}T\pi n$, are

$$K_\pm(\omega_n) = \frac{\pi}{2} \left[\mathcal{G}(\omega_n; x_{\rm b}, x_{\rm b}) \pm \mathcal{G}(\omega_n; x_{\rm b}^+, x_{\rm b}^-) \right]^{-1} . \quad (4)$$

The zero-frequency limit of K_- gives the charging energy, $E_{\rm c} = \hbar K_-(\omega_n \to 0)/2$. The dissipative effects of the collective modes are described by the spectral density

$$J_{\rm tot}(\omega) = \frac{1}{\pi} {\rm Im}[K_-(\omega_n \to +i\omega) + K_+(\omega_n \to +i\omega)]. \quad (5)$$

These quantities depend on the time ordered propagator $\mathcal{G}(\tau; x, x') \equiv \langle \mathbf{T}_\tau \vartheta(x,\tau)\vartheta(x',0)\rangle$ defined with respect to H_0 via the resolvent equation

$$\left[\frac{\omega_n^2}{v_{\rm F}} - \frac{\partial}{\partial x} \frac{v_{\rm F}}{g^2(x)} \frac{\partial}{\partial x} \right] \mathcal{G}(\omega_n; x, x') = \delta(x - x'). \quad (6)$$

We solve (6) linearly in $\varphi(x)$ ($x > x'$, $\omega_n > 0$),

$$\mathcal{G}(\omega_n; x, x') = \frac{g_\infty}{2\omega_n} \Big\{ e^{-\eta|x-x'|} - C \Big[e^{-\eta(x+x')} \times \\ \times \int_{-\infty}^{x'} {\rm d}y\, \varphi(y) e^{2\eta y} + e^{\eta(x+x')} \int_x^\infty {\rm d}y\, \varphi(y) e^{-2\eta y} \\ - e^{-\eta(x-x')} \int_{x'}^{x} {\rm d}y\, \varphi(y) \Big] \Big\} \quad (7)$$

with the constants $\eta = g_\infty \omega_n / v_{\rm F}$ and $C = g_\infty^3 \omega_n / 2\pi\hbar v_{\rm F}^2$.

Without impurities, one obtains from the above Green function the linear dc-conductance $G_0 = 2e^2\omega_n \mathcal{G}(\omega_n \to 0)/h = g_\infty e^2/h$, consistent with earlier work [3], and indicating that the linear conductance is a *global probe*. For the quantum dot, the results are more complex. On the one hand, by expanding (4) and (7) consistently to the order $a/L^* \ll 1$, the charging energy turns out to be $E_{\rm c} = \pi\hbar v_{\rm F}/2a g_{\rm b}^2$. It is determined by the interaction at the position of the quantum dot and is a *local probe*. On the other hand, the spectral density (5) can be decomposed in two parts that describe the influences of the leads, and of the energy discretization in the dot, $J_{\rm tot}(\omega) = J_{\rm l}(\omega) + J_{\rm d}(\omega) = J(\omega)[1 + \varepsilon \sum_{n=1}^\infty \delta(\hbar\omega - n\varepsilon)]$, where $J(\omega) = {\rm Im}[\mathcal{G}^{-1}(i\omega; x_{\rm b}, x_{\rm b})]/2$ and $\varepsilon \equiv 2 g_{\rm b} E_{\rm c}$ the discretization energy of the plasmon modes in the quantum dot. The function $J(\omega)$ is shown in Fig. 1 for $\varphi(x) = \exp[-(2x/L^*)^2]$; $[1 + (2x/L^*)^2]^{-1}$; and $[1 + \cos(\pi x/L^*)]/2$, $|x| < L^*$. There is a crossover between low- and high-frequency behavior, independently of the particular interaction, namely $J(\omega \ll \omega^*) = \omega/g_\infty$ and $J(\omega \gg \omega^*) = \omega/g_{\rm b}$, where $\omega^* = v_{\rm F}/g_\infty L^*$ is the crossover frequency corresponding to the characteristic length scale of the inhomogeneity.

The conductance $G_{\rm d}$ in the region of the Coulomb blockade is calculated by using sequential tunneling through high barriers in the presence of an external gate voltage $V_{\rm g}$ [10,11]. The latter defines the reference particle number in the dot. We consider $k_{\rm B}T \ll \varepsilon$. The dependence on the gate energy $\mu = e(V_{\rm g} - V_{\rm g}^{\rm res})$, that measures the shift with respect to the resonance value $eV_{\rm g}^{\rm res} = E_{\rm c}[2(n - n_0) + 1]$, and the temperature is

$$G_{\rm d} = \frac{e^2\, e^{-\mu/2k_{\rm B}T}}{4k_{\rm B}T \cosh(\mu/2k_{\rm B}T)}\, w_0(\varepsilon, T)\, \gamma(\mu, T). \quad (8)$$

Here $\gamma(\mu, T) = (\Delta^2/4) \int {\rm d}t\, \exp[i\mu t/\hbar - W_{\rm l}(t)]$ is a tunneling rate through a single impurity with tunneling frequency Δ. The temperature and time dependent dissipative kernel $W_{\rm l}(t)$ is given as an integral that contains the above spectral density $J(\omega)$, with a frequency cutoff $\omega_{\rm c}$. The weight $w_0(\varepsilon, T) = (\varepsilon/h) \int_0^{h/\varepsilon} {\rm d}t\, \exp[-W_{\rm d}(t)]$ is the time average of the periodic kernel of the dot $W_{\rm d}(t)$ [12]. Higher components that exhibit the excitation spectra do not contribute in the linear regime since $k_{\rm B}T < \varepsilon$. Below we present numerical results obtained for the Lorentzian inhomogeneity (Fig. 1).

The conductance for a single Coulomb blockade peak as a function of temperature and gate energy is shown in Fig. 2 for $g_\infty = 0.6$ and $g_{\rm b} = 0.3$. Independently of the inhomogeneity, the width w of the peak increases linearly with temperature. This implies that even in the inhomogeneous case the area A under the peak and the peak height are connected by $A \propto T G_{\rm d}^{\rm max}$. The maximum of the peak has a *non-monotonous* behavior in temperature with a minimum around the crossover that corresponds to the saddle in the 3D plot. The temperature dependence of the maximum can be written as $G_{\rm d}^{\rm max}(T) \propto T^{1/\tilde{g}(T) - 2}$ where $\tilde{g}(T)$ shows a crossover between $\tilde{g}(T \ll T^*) = g_\infty$ and $\tilde{g}(T \gg T^*) = g_{\rm b}$ with $k_{\rm B}T^* = \hbar\omega^*$. The non-monotonous behavior of the conductance is due to the particular choice of the interaction parameters $g_{\rm b} < 1/2 < g_\infty$. If both are larger or smaller than the critical value 1/2 the conductance decreases or increases monotonously with temperature, respectively. However, independently of the details near the crossover, the high- and low-temperature behaviors of the peaks are always dominated by the local and global properties of the interaction represented by $g_{\rm b}$ and g_∞, respectively. This means that a measurement of the linear conductance of the 1D quantum dot at low temperatures reflects

the interaction far away from the barrier, and therefore is a *global probe*. On the other hand, when measuring the Coulomb peak at higher temperatures, $T^* < T$, the interaction close to the dot will dominate. In this region, the experiment is a *local probe* for the interaction.

That contacts may become crucial when interpreting experimental data can also be seen by considering a TLL model with homogeneous interaction, $g(x) = g$, but its ends connected to a normal metallic lead via point like tunnel contacts. The temperature dependence of the contact conductance is [14]

$$G_{\rm c}(T) = \frac{1}{R_{\rm c}} \left(\frac{k_{\rm B}T}{\varepsilon} \right)^{\alpha} , \qquad \alpha = g^{-1} - 1 \tag{9}$$

with the prefactor $R_{\rm c}^{-1}$ containing the tunneling resistance between the lead and the TLL. For later comparison with the dot conductance we have chosen the discretization energy ε as the energy scale, including in $R_{\rm c}$ the rescaling between $\omega_{\rm c}$ and ε. If a quantum dot is embedded into the TLL with complete momentum randomization in the contacts, the total resistance can be obtained by adding the resistances of the contacts (9), and that of the quantum dot (8) with g = const: $G^{-1}(T) \equiv R(T) = G_{\rm c}^{-1}(T) + G_{\rm d}^{-1}(T)$. This can still yield Coulomb blockade oscillations (Fig. 3) but with a crossover in the temperature behavior. Near the maximum of a Coulomb peak the dot conductance scales as

$$G_{\rm d}^{\rm max}(T) = \frac{1}{R_{\rm d}} \left(\frac{k_{\rm B}T}{\varepsilon} \right)^{\alpha-1} . \tag{10}$$

In this region, the $G(T)$ will be determined by $G_{\rm c}(T)$ for temperatures lower than a crossover value that depends on the ratio $R_{\rm c}/R_{\rm d}$. On the other hand, in the tails of the peak, the $G_{\rm d}(T)$ always dominates, since there in any case $G_{\rm d}(T) \ll G_{\rm c}(T)$. The peak height $G^{\rm max}(T)$ shows a crossover between global (small T) and local (large T) power laws. For $\alpha < 1$, it has a maximum near the crossover temperature is $T^* = 2R_{\rm c}\varepsilon/R_{\rm d}k_{\rm B}$. For $R_{\rm c} = 0$ (no contact) only the power law behavior of the dot is obtained (black curve in Fig. 3).

The above results suggest that inhomogeneities cannot be neglected when deducing the interaction parameter in experimental data, especially in the presence of backscattering impurities. In [9] the charging energy of a 1D dot embedded in a CEO-quantum wire has been determined from the distance between Coulomb peaks, $E_{\rm c} = 2.2\,$meV with an estimate of $a \approx 100 \ldots 200\,$nm. It has been found that $g_{\rm b} \approx 0.4$ [12]. Measuring the temperature dependence of two conductance peaks in the range $T \approx 0.2 \ldots 2\,$K, a value $g_{\rm exp} \approx 0.8$ has been estimated (without spin) [9]. Taking into account the spin reduces the latter value slightly to ≈ 0.7 but does not solve the inconsistency. Given the good fit of the experimental data for $T < 2\,$K to the power law with a single value for $g_{\rm exp} > g_{\rm b}$ we conclude $g_{\rm exp} \approx g_{\infty}$, and that T^* should be larger than 2 K. By inspection of Fig. 2 we estimate $T^* \approx 10\,$K. This value certainly depends on the shape of the inhomogeneity, but should be of the correct order. With $v_{\rm F} \approx 10^5\,$m/s we find $L^* \approx 100\,$nm. This is not larger than the estimated length of the 1D quantum dot. But in view of the above idealized model assumptions, we can conclude that the temperature dependence of the conductance peaks are described by g_{∞}, and not by $g_{\rm b}$. Our result could be further experimentally addressed by changing the parameters of the CEO-quantum wires, which should influence especially L^*.

The influence of the contacts must also be present in the temperature behavior of the Coulomb blockade peaks of a quantum dot created by two buckles in a CN. In the absence of the dot a CN with ends attached to Au-wires shows the contact conductance $G_{\rm c}(T)$ of (9) with $\alpha = (g_{\rm nano}^{-1} - 1)/4$ and $g_{\rm nano} \approx 0.28$ [15]. The different relation between α and $g_{\rm nano}$ from the one given in (9) is due to the presence of further three non-interacting modes. In the presence of the dot, with the experimental value $\alpha = 0.68$ (corresponding to $g_{\rm nano} = 0.27$ [13]) we obtain from (10) that $G_{\rm d}^{\rm max}(T)$ increases with decreasing T (black curve in Fig. 3). On the other hand, the maximum $G^{\rm max}$ of the *total* conductance including the contacts decreases with decreasing T according to the power law of the contact conductance for $T < T^*$ and for a suitable ratio $R_{\rm c}/R_{\rm d}$. This is consistent with the experimental data and explains the discrepant behavior between the observed $G_{\rm max}(T)$ and the dot conductance in the sequential tunneling regime [13].

In conclusion, we have investigated transport in a 1D quantum dot embedded in a TLL with an inhomogeneity either due to non-uniform interaction or to the presence of contacts. We have identified quantities that measure the interaction locally and globally. We predict a crossover in the temperature behavior of Coulomb blockade peaks between regions that probe the interaction far away from and close to the dot for $T \ll T^*$, and $T \gg T^*$, respectively, though the Coulomb blockade remains intact. For the non-homogeneous interaction, the crossover is determined by the characteristic length of the region where $g(x) \approx$ const $\equiv g(x_{\rm b})$. When assuming a homogeneous interaction strength, the crossover is determined by the ratio between contact and dot resistances. Quite generally, the behavior can be understood by considering the energy scales set by the quantum dot and the inhomogeneity: if $E_{\rm c} \gg k_{\rm B}T^*$ one may fulfill the condition $k_{\rm B}T \ll E_{\rm c}$ necessary for getting Coulomb blockade and nevertheless observe a crossover from global to local behavior in the non-analytic power law dependence of the conductance peaks which is governed by the energy scale of the inhomogeneity. The results are used to understand consistently and quantitatively recent, fundamentally important experiments done on semiconductor CEO-quantum wires and CNs.

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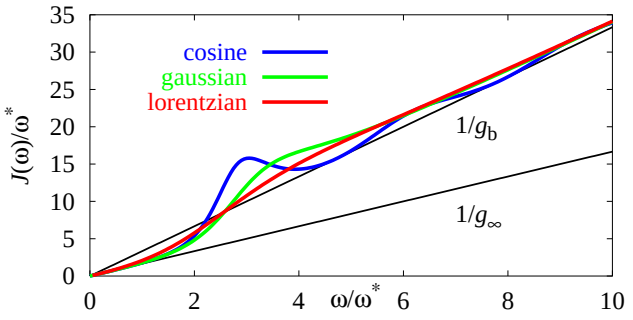

FIG. 1. The spectral density $J(\omega)/\omega^*$ for different interactions centered around $x_b = 0$ (see text). Black lines: asymptotic behaviors with slopes given by $g_\infty = 0.6$ and $g_b = 0.3$.

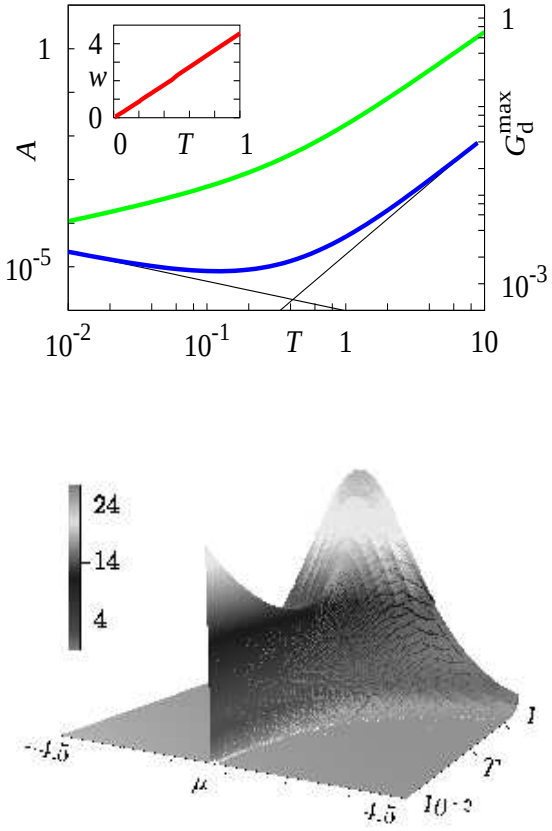

FIG. 2. Conductance peak of a 1D quantum dot for lorentzian interaction ($x_b = 0$, $g_\infty = 0.6$, $g_b = 0.3$, $\omega^*/\omega_c = 10^{-3}$). Top: double logarithmic plot as a function of the temperature T in units of $T^* = \mu^*/k_B$ ($\mu^* = \hbar\omega^*$) of the peak height G_d^{max} (blue curve, right scale, units $G_0 = (\Delta/4\omega_c)^2(\varepsilon/\hbar\omega_c)^{1/g_b}e^2/\hbar$), and the peak area A normalized to $G_0\mu^*$ (green curve, left scale); inset: peak width W (units μ^*) as a function of T. Bottom: conductance as a function of the gate energy μ and of the temperature T (units μ^*, T^*), color code (left, units $10^{-4}G_0$).

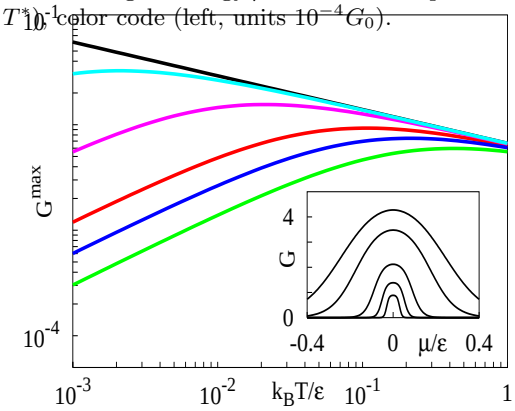

FIG. 3. Double logarithmic plot of the height of the conductance peak of a quantum dot (units e^2/h) connected to leads via resistive tunnel contacts as a function of temperature k_BT/ε with $2R_c/R_d = 0, 10^{-3}, 10^{-2}, 5{\cdot}10^{-2}, 10^{-1}, 2{\cdot}10^{-1}$ (top to bottom), $\alpha \equiv -1 + 1/g = 0.68$ and $R_d = 150h/e^2$. Inset: conductance G in units of $10^{-3}e^2/h$ for $R_c/R_d = 0.1$ (green curve of main figure) as a function of μ/ε; temperatures $10^2T/\varepsilon = 8, 5, 2, 1, 0.5$ (top to bottom).